Propagators of hot SU(2) gauge theory from 3d adjoint Higgs model *

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We study propagators of the lattice 3d adjoint Higgs model, considered as an effective theory of 4d $SU(2)$ gauge theory at high temperature. The propagators are calculated in so-called λ-gauges. From the long distance behaviour of the propagators we extract the screening masses. It is shown that the pole masses extracted from the propagators agree well with the screening masses obtained recently in finite temperature $SU(2)$ theory. The gauge dependence of the screening masses is also discussed.

1. INTRODUCTION

The screening of static chromo-electric fields is one of the most outstanding properties of QCD at finite temperature and its investigation is important both from a theoretical and phenomenological point of view (for phenomenological applications see e.g. [1]). In leading order of perturbation theory the associated inverse screening length (Debye mass) is defined as the IR limit of the longitudinal part of the gluon self energy $\Pi(k_0 = 0, \mathbf{k} \to 0)$. However, as the screening phenomenon is related to the long distance behaviour of QCD the naive perturbative definition of the Debye mass is obstructed by severe IR divergences of thermal field theory and beyond leading order the above definition is no longer applicable. Rebhan has shown that the definition of the Debye mass through the pole of the longitudinal part of the gluon propagator is gauge invariant [2]. However, this definition requires the introduction of a so-called magnetic screening mass, a concept introduced long ago [3] to cure the IR singularities of finite temperature non-Abelian theories. Analogously to the electric (Debye) mass the magnetic mass can be defined as a pole of the transverse part of the finite temperature gluon propagator. Because of the IR sensitivity a non-perturbative determination of the screening mass is necessary.

The main question which we will try to clarify in this contribution is whether the screening masses, defined as poles of the corresponding lattice propagators in Landau gauge can be determined from the effective theory (Section 2). We will consider the simplest case of the $SU(2)$ gauge group, where precise 4d data on screening masses exist for a huge temperature range [4]. We will also discuss the question of gauge dependence of the screening masses (Section 3).

2. SCREENING MASSES FROM 3D ADJOINT HIGGS MODEL

The lattice action for the 3d adjoint Higgs model used in the present paper has the form

$$
\begin{aligned}
S = {} & \beta \sum_P \frac{1}{2} Tr U_P + \\
& \beta \sum_{\mathbf{x},\hat{i}} \frac{1}{2} Tr A_0(\mathbf{x}) U_i(\mathbf{x}) A_0(\mathbf{x}+\hat{i}) U_i^\dagger(\mathbf{x}) + \\
& \sum_{\mathbf{x}} \left[-\beta \left(3 + \frac{1}{2} h \right) \frac{1}{2} Tr A_0^2(\mathbf{x}) + \right. \\
& \left. \beta x \left(\frac{1}{2} Tr A_0^2(\mathbf{x}) \right)^2 \right], \qquad (1)
\end{aligned}
$$

where U_P is the plaquette, U_i are the usual link variables and the adjoint Higgs field is parameterized by anti-hermitian matrices $A_0 = i \sum_a \sigma^a A_0^a$ (σ^a are the usual Pauli matrices). Furthermore β is the lattice gauge coupling, x parametrizes the quartic self coupling of the Higgs field and h denotes the bare Higgs mass squared. This

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model is known to have two phases: the broken (Higgs) phase and the symmetric (confinement) phase separated by a line of 1^{st} order phase transitions [5,6]. The high temperature phase of the 4d $SU(2)$ gauge theory corresponds to some surface in the parameter space (β, h, x), the surface of 4d physics $h = h_{4d}(x, \beta)$. This surface may lie in the symmetric phase or in the broken phase, i.e. the physical phase might be either the symmetric or the broken phase. The surface of 4d physics determined by the 2-loop level dimensional reduction [6] lies in the broken phase. This fact appears to be self-contradictory because if $g \ll 1$ for expectation value of A_0 one has $A_0 \sim 1/g$ but the dimensional reduction is only valid if $A_0 \ll \pi T$. To overcome this difficulty of the dimensional reduction the authors of Ref. [6] suggested to use the 1^{st} order nature of the phase transition and perform measurements at the values of the parameters obtained from the 2-loop dimensional reduction but in the metastable phase [2]. The obvious problem with this approach is that the metastable phase disappears in the infinite volume limit. Therefore in Ref. [7] simulations were done in the symmetric phase and it was suggested to determine the surface of 4d physics by non-perturbative matching.

In this section we are going to review our results on electric and magnetic screening masses obtained from the Landau gauge propagators in the symmetric phase as well as in the metastable region of our finite lattices. Most of our numerical studies have been performed on lattices of size $32^2 \times 64$ and at $\beta = 16$. Simulations in the metastable region have been performed at the values of the parameters obtained from 2-loop dimensional reduction [6]. The two sets of values of h and x used in our simulations in the symmetric phase are shown in Table 1, where also the corresponding temperature values as well as the values of h corresponding to the 2-loop dimensional reduction (h_{4d}) are indicated. The temperature scale is essentially fixed by x. The detailed procedure of choosing the parameters in the symmetric phase is given in Ref. [7]. The results on the screening masses are summarized in Figure 1 where also the results of 4d simulations [4] are shown.

[2] In a finite volume there always is a region in the parameter space where the symmetric and broken phases are metastable. Only in the infinite volume limit an assignement to one of these phases will become possible. In fact, on lattices typically used in numerical calculations the surface of 4d physics lies in this metastable region [6].

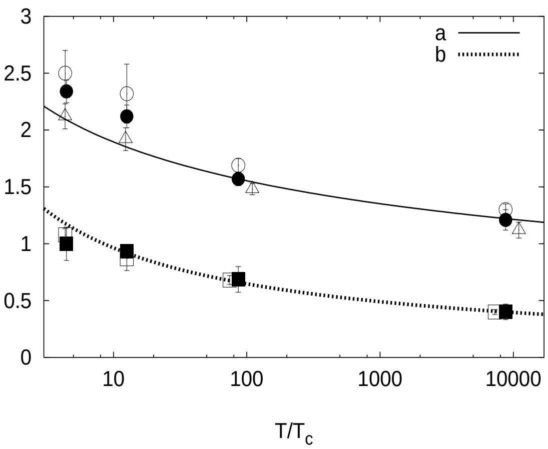

Figure 1. The screening masses in units of the temperature. Shown are the Debye mass m_D for the first (filled circles) and the second (open circles) set of h, and the magnetic mass m_T for the first (filled squares) and the second (open squares) set of h. The line (a) and line (b) represent the fit for the temperature dependence of the Debye and the magnetic mass from 4d simulations The open triangles are the values of the Debye mass for $h_{4d}(x, \beta)$ obtained from the 2-loop dimensional reduction in the metastable region. Some data points at the temperature $T \sim 90T_c$ and $T \sim 9000T_c$ have been shifted in the temperature scale for better visibility.

Table 1
The two sets of the bare mass squared used in the simulation and those which correspond to the 2-loop dimensional reduction for $\beta = 16$

x	T/T_c	h_I	h_{II}	h_{4d}
0.09	4.433	-0.2652	-0.2622	-0.2700
0.07	12.57	-0.2528	-0.2490	-0.2588
0.05	86.36	-0.2365	-0.2314	-0.2437
0.03	8761	-0.2085	-0.2006	-0.2181

As one can see from the figure the agreement between the masses obtained from 4d and 3d simulation is rather good. The magnetic mass practically shows no dependence on h and its value is rather close to the magnetic mass of 3d pure gauge theory $m_T = 0.46(3)g_3^2$ [7] (g_3^2 is the 3d gauge coupling). The electric mass shows some dependence on h. For relatively low temperatures ($T < 50T_c$) the best agreement with the 4d data for the Debye mass is obtained for values of h corresponding to 2-loop dimensional reduction and lying in the metastable region. For higher temperatures, however, practically no distinction can be made between the three choices of h.

3. GAUGE DEPENDENCE OF THE SCREENING MASSES

Let us turn to the discussion of the gauge dependence of the propagator masses. The propagator pole mass was proven to be gauge independent at any given order of perturbation theory [8]. Whether this holds also non-perturbatively is, however, an open question [3]. To study the gauge dependence of the pole masses we have used the so-called λ-gauges [9] defined by the gauge fixing condition

$$\lambda\partial_3 A_3 + \partial_2 A_2 + \partial_1 A_1 = 0. \qquad (2)$$

The case $\lambda = 1$ corresponds to the Landau gauge. For the numerical analysis the following values of the parameter λ have been chosen: $\lambda = 0.5,\ 1,\ 2,\ 8$. We have measured the electric and the magnetic correlators on a $32^2 \times 96$ lattice at $x = 0.03$ and two values of β and h: $\beta = 16,\ h = -0.2085$ and $\beta = 24,\ h = -0.1510$ both corresponding to the symmetric phase. The results of these measurements for the electric (A_0) propagator are shown in Figure 2. As one can see from the Figure the large distance behaviour of the propagators seems to exhibit some gauge dependence which becomes visible for large values of λ. The situation is similar for the magnetic propagator [10]. To what extent this behaviour is

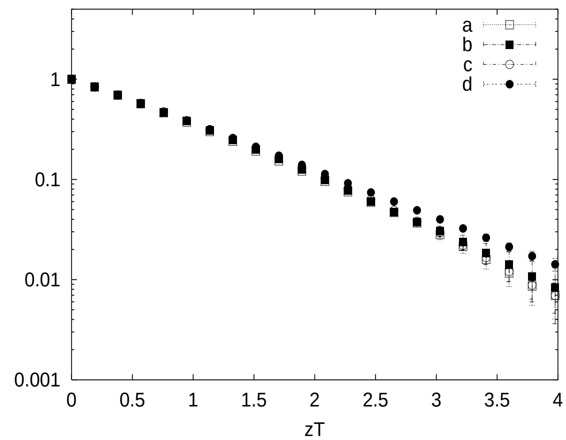

Figure 2. The electric (A_0) propagators at $x = 0.03,\ h = -0.1510,\ \beta = 24$ for different values of λ: $\lambda = 0.5$ (a), 1.0 (b), 2.0 (c), 8.0 (d),

influenced by the gauge fixing procedure used by us requires further analysis. Here one also should consider gauges other than λ-gauges [11].

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[3]The propagator pole in the 3d adjoint Higgs model does not correspond to an asymptotic state (the theory is confining) and therefore there there is a priori no reason for gauge independence of the pole mass.